Epistemic "Holes" in Space-Time

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A number of models of general relativity seem to contain "holes" that are thought to be "physically unreasonable." One seeks a condition to rule out these models. We examine a number of possibilities already in use. We then introduce a new condition: epistemic hole-freeness. Epistemic hole-freeness is not just a new condition—it is new in kind. In particular, it does not presuppose a distinction between space-times that are "physically reasonable" and those that are not.

1. Introduction. A number of models of general relativity seem to contain "holes" that are thought to be "physically unreasonable." One seeks a condition to rule out these models. We examine a number of possibilities already in use. We then introduce a new condition: epistemic hole-freeness. Epistemic hole-freeness is not just a new condition—it is new in kind. In particular, it does not presuppose a distinction between space-times that are "physically reasonable" and those that are not.

2. Preliminaries. We begin with a few preliminaries concerning the relevant background formalism of general relativity.[1] An n-dimensional, relativistic *space-time* (for $n \geq 2$) is a pair of mathematical objects (M, g_{ab}). Object M is a connected n-dimensional manifold (without boundary) that is smooth (infinitely differentiable). Here, g_{ab} is a smooth, nondegenerate, pseudo-Riemannian metric of Lorentz signature $(+, -, \ldots, -)$ defined on M. Note that M is assumed to be *Hausdorff*; for any distinct $p, q \in M$, one

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1. The reader is encouraged to consult Hawking and Ellis (1973), Wald (1984), and Malament (2012) for details. An outstanding (and less technical) survey of the global structure of space-time is given by Geroch and Horowitz (1979).

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can find disjoint open sets O_p and O_q containing p and q, respectively. We say two space-times (M, g_{ab}) and (M', g'_{ab}) are *isometric* if there is a diffeomorphism $\varphi : M \to M'$ such that $\varphi_*(g_{ab}) = g'_{ab}$.

For each point $p \in M$, the metric assigns a cone structure to the tangent space M_p. Any tangent vector ξ^a in M_p will be *time-like* if $g_{ab}\xi^a\xi^b > 0$, *null* if $g_{ab}\xi^a\xi^b = 0$, or *space-like* if $g_{ab}\xi^a\xi^b < 0$. Null vectors create the cone structure; time-like vectors are inside the cone, while space-like vectors are outside. A *time orientable* space-time is one that has a continuous time-like vector field on M. A time orientable space-time allows one to distinguish between the future and past lobes of the light cone. In what follows, it is assumed that space-times are time orientable.

For some open (connected) interval $I \subseteq \mathbb{R}$, a smooth curve $\gamma : I \to M$ is *time-like* if the tangent vector ξ^a at each point in $\gamma[I]$ is time-like. Similarly, a curve is *null* (respectively, *space-like*) if its tangent vector at each point is null (respectively, space-like). A curve is *causal* if its tangent vector at each point is either null or time-like. A causal curve is *future directed* if its tangent vector at each point falls in or on the future lobe of the light cone.

An *extension* of a curve $\gamma : I \to M$ is a curve $\gamma' : I' \to M$ such that I is a proper subset of I' and $\gamma(s) = \gamma'(s)$ for all $s \in I$. A curve is *maximal* if it has no extension. A curve $\gamma : I \to M$ in a space-time (M, g_{ab}) is a *geodesic* if $\xi^a \nabla_a \xi^b = 0$, where ξ^a is the tangent vector and ∇_a is the unique derivative operator compatible with g_{ab}. Let $\gamma : I \to M$ be a time-like curve with unit tangent vector ξ^b. The acceleration vector is $\alpha^b = \xi^a \nabla_a \xi^b$, and the magnitude of acceleration is $a = (-\alpha^b \alpha_b)^{1/2}$. The *total acceleration* of γ is $\int_\gamma a\,ds$, where s is elapsed proper time along γ.

For any two points $p, q \in M$, we write $p \ll q$ if there exists a future directed time-like curve from p to q. We write $p < q$ if there exists a future directed causal curve from p to q. These relations allow us to define the *time-like and causal pasts and futures* of a point p: $I^-(p) = \{q : q \ll p\}$, $I^+(p) = \{q : p \ll q\}$, $J^-(p) = \{q : q < p\}$, and $J^+(p) = \{q : p < q\}$. Naturally, for any set $S \subseteq M$, define $J^+[S]$ to be the set $\cup\{J^+(x) : x \in S\}$, and so on. A set $S \subset M$ is *achronal* if $S \cap I^-[S] = \varnothing$. A space-time satisfies *chronology* if, for each $p \in M$, $p \notin I^-(p)$.

A point $p \in M$ is a *future endpoint* of a future directed causal curve $\gamma : I \to M$ if, for every neighborhood O of p, there exists a point $t_0 \in I$ such that $\gamma(t) \in O$ for all $t > t_0$. A *past endpoint* is defined similarly. A causal curve is *future inextendible* (respectively, *past inextendible*) if it has no future (respectively, past) endpoint.

For any set $S \subseteq M$, we define the *past domain of dependence of S*, written $D^-(S)$, to be the set of points $p \in M$ such that every causal curve with past endpoint p and no future endpoint intersects S. The *future domain of dependence of S*, written $D^+(S)$, is defined analogously. The entire *domain of dependence of S*, written $D(S)$, is just the set $D^-(S) \cup D^+(S)$. The *edge* of an achronal set $S \subset M$ is the collection of points $p \in S$ such that every open neighborhood O of p contains a point $q \in I^+(p)$, a point $r \in I^-(p)$, and a time-like curve from r to q that does not intersect S. A set $S \subset M$ is a *slice* if it is closed, achronal, and without edge. A space-time (M, g_{ab}) that contains a slice S such that $D(S) = M$ is said to be *globally hyperbolic*.

3. A Condition to Disallow Holes? Consider the following example (see fig. 1).

> **Example 1.** Let (M, g_{ab}) be Minkowski space-time and let p be any point in M. Consider the space-time $(M - \{p\}, g_{ab})$.

The space-time seems to have an artificial "hole." One seeks to find a (simple, physically meaningful) condition to disallow the example. (The condition need not be a sufficient condition for "physical reasonableness"; it need only be necessary.) But "although one perhaps has a good intuitive idea of what it is that one wants to avoid, it seems to be difficult to formulate a precise condition to rule out such examples" (Geroch and Horowitz 1979, 275).

Many of the conditions used to rule out the "hole" in example 1 require that certain regions of (or curves in) space-time be "as large as they can be." For example, geodesic completeness requires every geodesic to be as large as it can be in a certain sense. Hole-freeness essentially requires the domain of dependence of every space-like surface to be as large as it can be. Inextendibility requires the entirety of space-time to be as large as it can be. Let us examine each of these three conditions in more detail. First, consider geodesic completeness.

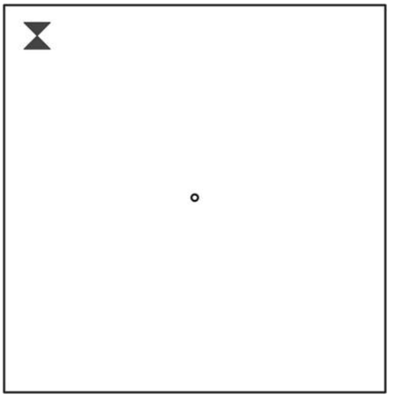

Figure 1. Minkowski space-time with a point removed from the manifold.

Definition. A space-time (M, g_{ab}) is *geodesically complete* if every maximal geodesic $\gamma : I \to M$ is such that $I = \mathbb{R}$. A space-time is *geodesically incomplete* if it is not geodesically complete.

If an incomplete geodesic is time-like or null, there is a useful distinction one can introduce (which we will need later on). We say that a future directed time-like or null geodesic $\gamma : I \to M$ without a future endpoint is *future incomplete* if there is an $r \in \mathbb{R}$ such that $s < r$ for all $s \in I$. A *past incomplete* time-like or null geodesic is defined analogously. Next, consider inextendibility.

Definition. A space-time (M, g_{ab}) is *extendible* if there exists a space-time (M', g'_{ab}) and an isometric embedding $\varphi : M \to M'$ such that $\varphi[M]$ is a proper subset of M'. Here, the space-time (M', g'_{ab}) is an *extension* of (M, g_{ab}). A space-time is *inextendible* if it has no extension.

Finally, consider hole-freeness. Initially, one defined (Geroch 1977) a space-time (M, g_{ab}) to be *hole-free* if, for every space-like surface $S \subset M$ and every isometric embedding $\varphi : D(S) \to M'$ into some other space-time (M', g'_{ab}), we have $\varphi(D(S)) = D(\varphi(S))$. The definition seemed to be satisfactory. But surprisingly, it turns out the definition is too strong; Minkowski space-time fails to be hole-free under this formulation (Krasnikov 2009). But one can make modifications to avoid this consequence (Manchak 2009).

Let (K, g_{ab}) be a globally hyperbolic space-time. Let $\varphi : K \to K'$ be an isometric embedding into a space-time (K', g'_{ab}). We say (K', g'_{ab}) is an *effective extension* of (K, g_{ab}) if, for some Cauchy surface S in (K, g_{ab}), $\varphi[K] \subsetneq \text{int}\,(D(\varphi[S]))$ and $\varphi[S]$ is achronal. Hole-freeness can then be defined as follows.

Definition. A space-time (M, g_{ab}) is *hole-free* if, for every set $K \subseteq M$ such that $(K, g_{ab|K})$ is a globally hyperbolic space-time with Cauchy surface S, if $(K', g_{ab|K'})$ is not an effective extension of $(K, g_{ab|K})$ where $K' = \text{int}\,(D(S))$, then there is no effective extension of $(K, g_{ab|K})$.

What is the relationship between the three conditions? There are only two implication relations between them (Manchak 2014).

Proposition 1. Any space-time that is geodesically complete is hole-free and inextendible.

Now, any of the three conditions can be used to rule out the "hole" in example 1. But due to the singularity theorems (Hawking and Penrose 1970), geodesic completeness is now considered to be much too strong a condition; it seems to be violated by "physically reasonable" space-times. In what follows, let us focus on the remaining two conditions that are usually taken to be satisfied by all "physically reasonable" space-times. Indeed, these two conditions are still in use (see Earman 1995). Might hole-freeness or inextendibility (or their conjunction) be the condition we are looking for? Consider the following example.

Example 2. Let (M, g_{ab}) be Minkowski space-time, and let p be any point in M. Let $\Omega : M - \{p\} \rightarrow \mathbb{R}$ be a smooth positive function that approaches zero as the point p is approached. Now consider the space-time $(M - \{p\}, \Omega^2 g_{ab})$.

The space-time in example 2 is inextendible and hole-free. Nonetheless, it seems there is still an artificial "hole" in the space-time. One seeks a (simple, physically meaningful) condition to rule out even these holes.

4. A New Condition. Consider the following definition form.

Definition. A space-time (M, g_{ab}) has an *epistemic hole* if there are two future inextendible time-like curves γ and γ' with the same past endpoint and which ____ such that $I^-[\gamma]$ is a proper subset of $I^-[\gamma']$.

The physical significance of the definition form is this: Suppose two observers are both present at some event. Now suppose (subject to the restrictions in the blank) they go their separate ways. If it is the case that one observer can eventually know everything the other can eventually know and more, then there is a kind of epistemic "hole" preventing the latter observer from knowing the extra bit. One might require the region of space-time that an observer can eventually know to be "as large as it can be." In other words, one might require space-time to be free of epistemic holes.

If no restrictions are given in the blank, examples 1 and 2 count as having epistemic holes as we would hope. But, unfortunately, this version of the condition is too strong; it rules out space-times that are usually thought to be "physically reasonable" in some sense. Take Minkowski space-time, for example. It counts as having epistemic holes. (Consider any point in the Minkowski space-time. Now consider any observer at the point who, with infinite total acceleration, reaches "null infinity" and another observer at the point who does not. See fig. 2.)

In order to not count Minkowski space-time as having epistemic holes, one seeks to fill the blank with reasonable restrictions. Let us consider two natural possibilities: "are geodesics" and "have finite total acceleration." Let EH(g) and EH(f) respectively denote these two versions of the epistemic

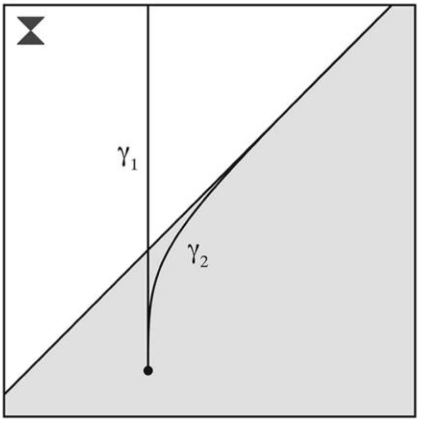

Figure 2. Observers γ_1 and γ_2 in Minkowski space-time. The set $I^-[\gamma_2]$ (the shaded area) is a proper subset of $I^-[\gamma_1]$ (the entire manifold).

hole definition. In addition, if a space-time fails to have an EH(g), let us say it is EH(g)-free (and respectively for the EH(f) case).

Clearly, if a space-time is EH(f)-free, then it also EH(g)-free.[2] And as we would hope, examples 1 and 2 each have an EH(g) and therefore an EH(f) (see fig. 3). Indeed, acausal examples aside, it seems almost every artificially mutilated space-time will have an epistemic hole of some type. Now, Minkowski space-time is EH(f)-free and EH(g)-free by construction. What about other "physically reasonable" space-times? The Schwarzchild solution is a good test case; its future inextendible time-like curves have event horizons that might allow for epistemic holes.[3] But this is not the case; it and its Kruskal extension count as EH(f)-free and EH(g)-free (see fig. 4).

One can also show that the de Sitter, anti–de Sitter, and Gödel models all count as both EH(f)-free and EH(g)-free. Misner space-time is neither EH(f)-free nor EH(g)-free (see fig. 5). However, Misner space-time harbors "naked singularities" thought to be physically unreasonable. Consider the following influential definition (Geroch and Horowitz 1979; Earman 1995).[4]

2. One wonders whether the converse is also true. We conjecture that, due to its peculiar time-like geodesic structure, Reissner-Nordström space-time is EH(g)-free but not EH(f)-free.

3. For a complete treatment of event horizons, see Rindler (1956).

4. The concept of terminal indecomposable past sets, which can be shown to be precisely the time-like pasts of future inextendible time-like curves, has been used to formulate a type of naked singularity definition. The definition turns out to be equivalent to the failure of global hyperbolicity (Penrose 1999). For details on the relationship between global hyperbolicity and epistemic holes, see below.

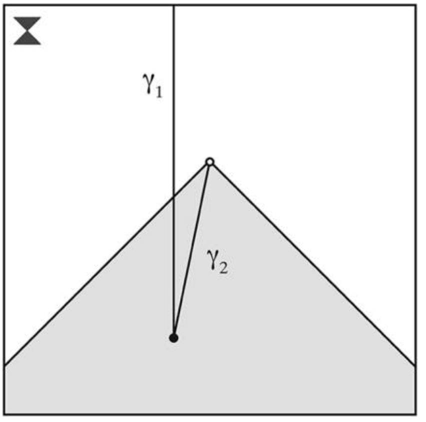

Figure 3. Geodesic observers γ_1 and γ_2 in Minkowski space-time with one point removed from the manifold. The set $I^-[\gamma_2]$ (the shaded area) is a proper subset of $I^-[\gamma_1]$ (the entire manifold).

Definition. A space-time (M, g_{ab}) is *nakedly singular* if there is a point $p \in M$ and a future incomplete time-like geodesic γ such that $\gamma \subset I^-(p)$.

What is the relationship between naked singularities and epistemic holes? Consider the following examples.

Example 3. Let (M, g_{ab}) be Minkowski space-time, and let p be any point in M. Let $\Omega : M - \{p\} \rightarrow \mathbb{R}$ be a smooth positive function that approaches infinity as the point p is approached. Now consider the space-time $(M - \{p\}, \Omega^2 g_{ab})$.

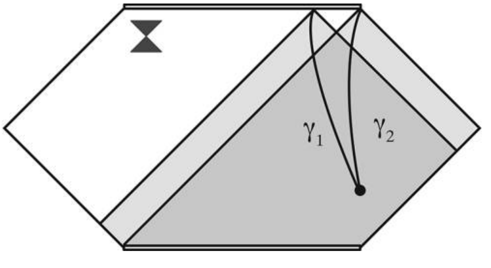

Figure 4. Conformal diagram of Kruskal-Schwarzchild space-time. Any observers γ_1 and γ_2 with finite total acceleration are such that if $I^-[\gamma_2]$ and $I^-[\gamma_1]$ (shaded areas) are distinct, then they do not fully overlap.

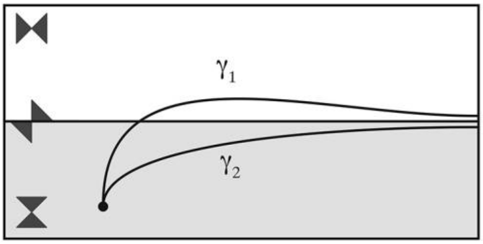

Figure 5. Unrolled Misner space-time. Geodesic observers γ_1 and γ_2 are such that the set $I^{-}[\gamma_2]$ (the shaded area) is a proper subset of $I^{-}[\gamma_1]$ (the entire manifold).

Example 4. Let (M, g_{ab}) be two-dimensional Minkowski space-time in standard *t*, *x* coordinates that is "rolled up" along the *t* direction. Let *p* be any point in *M*. Consider the space-time $(M - \{p\}, g_{ab})$.

Example 3 is geodesically complete (and therefore contains no naked singularities). But it has both an EH(f) and an EH(g). However, example 4 contains naked singularities but is EH(f)-free and EH(g)-free. Now, example 4 is not causally well behaved. If one were to limit attention to space-times satisfying chronology, can one still find examples with naked singularities and without epistemic holes? At least in the case of EH(f), no.[5]

Proposition 2. Any EH(f)-free, chronological space-time is not nakedly singular.

Proof. Let (M, g_{ab}) be a chronological, nakedly singular space-time. Let γ be a future incomplete time-like geodesic with past endpoint $q \in M$ such that $\gamma \subset I^{-}(p)$ for some $p \in M$. Let γ' be any time-like curve with finite total acceleration with past endpoint *q* that runs through *p* and is future inextendible. Clearly, $I^{-}[\gamma] \subseteq I^{-}[\gamma']$. Suppose $I^{-}[\gamma] = I^{-}[\gamma']$. Since $p \in I^{-}[\gamma']$, we have $p \in I^{-}[\gamma]$. So, there is a point $r \in \gamma$ such that $p \in I^{-}(r)$. But $r \in I^{-}(p)$. It follows that $p \in I^{-}(p)$, which is a violation of chronology: a contradiction. So, $I^{-}[\gamma] \neq I^{-}[\gamma']$. Thus, $I^{-}[\gamma]$ is a proper subset of $I^{-}[\gamma']$. So, there is an EH(f) in (M, g_{ab}). QED

The proposition shows that, if one takes EH(f)-freeness as a necessary condition of physical reasonableness, then the weak causality assumption of chronology rules out naked singularities. Contrast this result with one (Geroch and Horowitz 1979) which shows that the strong causality assumption of

5. It is an open question whether the result holds for the EH(g) case as well.

global hyperbolicity is, by itself, enough to exclude naked singularities.[6] Now, what is the relationship between global hyperbolicity and epistemic holes? Consider the following example.

> **Example 5.** Let (M, g_{ab}) be Minkowski space-time, and let p be any point in M. Let M' be the set $I^{-}(p)$. Let $\Omega : M' \to \mathbb{R}$ be a smooth positive function that approaches infinity as the boundary of $I^{-}(p)$ is approached. Now consider the space-time $(M', \Omega^2 g_{ab})$. (See fig. 6.)

Example 5 shows that a globally hyperbolic space-time, indeed even a globally hyperbolic space-time that is geodesically complete, can nonetheless fail to be EH(f)-free and EH(g)-free. However, example 4 shows that a space-time that is nonglobally hyperbolic, indeed even a nonchronological space-time that fails to be inextendible and hole-free, can nonetheless be EH(f)-free and EH(g)-free. In sum: epistemic holes are very different from "holes" and "singularities" of various kinds.

5. A New Kind of Condition. Stepping back, one may ask: what justifies the use of epistemic hole-freeness? As we have seen, the condition rules out many intuitively "physically unreasonable" space-times (including those, like example 2, that inextendibility and hole-freeness fail to rule out). However, no model with epistemic holes has yet been found that is clearly "physically reasonable." It is our position that this alone provides sufficient justification for the condition. In any case, the proper sorting of the intuitively "physically reasonable" and "physically unreasonable" examples is, at root, the justification for the widely used conditions of inextendibility and hole-freeness (Earman 1995; Manchak 2011). And, as with these other two conditions, one can hope to put epistemic hole-freeness to work in proving theorems of interest (such as proposition 2).

In addition, we wish to highlight an important way in which the condition of epistemic hole-freeness is far superior to the conditions of inextendibility and hole-freeness. The definitions of the latter two conditions make reference to (and are functions of) the entire class of relativistic space-times; they require that certain regions of space-time be "as large as they can be" in the sense that one compares them, from a God's eye point of view, to similar regions in all possible space-times. And whether a space-time counts as inextendible or hole-free depends crucially on the makeup of this class of all possible space-times. But what is this class? We would like to emphasize

6. We know from (the universal covering space of) anti–de Sitter space-time that global hyperbolicity is neither equivalent to the conjunction of chronology and EH(f)-freeness nor equivalent to the conjunction of chronology and EH(g)-freeness.

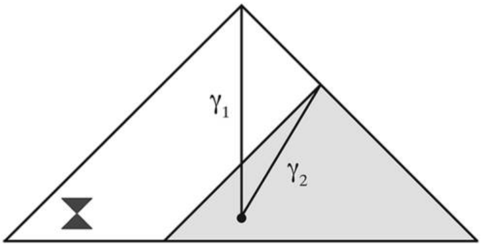

Figure 6. Geodesic observers γ_1 and γ_2 in a portion of conformal Minkowski space-time. The set $I^-[\gamma_2]$ (the shaded area) is a proper subset of $I^-[\gamma_1]$ (the entire manifold).

that whatever the answer winds up being depends on assumptions concerning what "physically reasonable" space-times are. And the fact that we have yet to pin down the class of "physically reasonable" space-times should give us pause. In essence, the fact implies that we have yet to pin down the class of inextendible space-times and the class of hole-free space-times.

An example might serve to illustrate the point. Following standard practice, we have assumed in the preceding that all manifolds are Hausdorff. Under this assumption, Minkowski space-time counts as inextendible. The space-time is "as large as it can be" in the sense that it cannot be properly and isometrically embedded into another space-time. But note that if the Hausdorff assumption is dropped, Minkowski space-time now counts as extendible (see fig. 7). The example shows that, whatever else is the case, the class of "possible" space-times, as standardly interpreted, is not a class

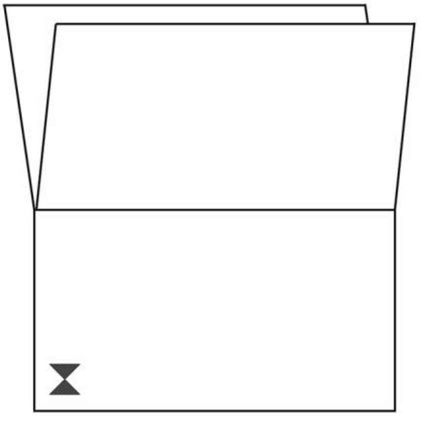

Figure 7. Non-Hausdorff extension of Minkowski space-time.

of merely logically or mathematically possible space-times; non-Hausdorff space-times are logically and mathematically well defined (Earman 2008), and Minkowski space-time is standardly interpreted as inextendible. Thus, the use of the condition of inextendibility has, all along, presupposed a distinction between space-times that are "physically reasonable" and those that are not. Once this important fact is clear, one is naturally "tempted to revise the principle of inextendibility" (Geroch 1970, 278). But how should one revise? Any revision is dubious given that we do not know, and arguably cannot know (Manchak 2011), the makeup of the class of "physically reasonable" space-times.

To see why this might be, consider another example: the bottom half of Misner space-time (see fig. 5). It is globally hyperbolic and counts as extendible in the preceding. But suppose the cosmic censorship conjecture of Penrose (1979) is correct, and all "physically reasonable" space-times are globally hyperbolic. Then the bottom half of Misner space-time now counts as inextendible. It cannot be embedded properly and isometrically into a globally hyperbolic space-time; the space-time is "as large as it can be" if the possibility space is limited in just the way some experts think it is. It follows that whether the bottom half of Misner space-time counts as inextendible depends crucially on the outcome of the cosmic censorship conjecture—a conjecture that is far from settled (Earman 1995; Penrose 1999). One is seemingly forced to conclude that the very content of the condition of inextendibility is unavoidably murky. (A similar argument can be given for the case of hole-freeness.)

Now consider epistemic hole-freeness. It requires that certain regions of space-time be "as large as they can be" in the sense that one compares them to similar regions within the very same space-time. The condition does not make reference to the class of all "possible" space-times. And thus, the condition does not presuppose that a distinction has been made between space-times that are "physically reasonable" and those that are not.[7] The content of the condition is perfectly clear. Among conditions used to rule out "holes" in space-time, epistemic hole-freeness appears to be alone in possessing this desirable quality.[8]

7. Some distinctions may need to be made to "get things off the ground," as it were. For example, in order to articulate the definition of epistemic hole-freeness, one must presuppose time orientability. But once off the ground, the definition of epistemic hole-freeness is not a function of the class of "physically reasonable" space-times (as are inextendibility and hole-freeness).

8. Many other familiar conditions also possess this quality, e.g., the Hausdorff condition, time orientability, chronology, the standard energy conditions. But these are not used to rule out "holes" in space-time.

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