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Time (hole?) machines

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ABSTRACT

Within the context of general relativity, we consider a type of "time machine" and introduce the related "hole machine". We review what is known about each and add results of our own. We conclude that (so far) the hole machine advocate is in a better position than the time machine advocate.

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1. Introduction

Within the context of general relativity, we consider a type of "time machine" and introduce the related "hole machine". We review what is known about each and add results of our own. We conclude that (so far) the hole machine advocate is in a better position than the time machine advocate.

2. Background structure

We begin with a few preliminaries concerning the relevant background formalism of general relativity.[1] An n-dimensional, relativistic *spacetime* (for $n \geq 2$) is a pair of mathematical objects (M, g_{ab}). M is a connected n-dimensional manifold (without boundary) that is smooth (infinitely differentiable). Here, g_{ab} is a smooth, non-degenerate, pseudo-Riemannian metric of Lorentz signature $(+,-,\ldots,-)$ defined on M. Each point in the manifold M represents an "event" in spacetime.

For each point $p \in M$, the metric assigns a cone structure to the tangent space M_p. Any tangent vector ξ^a in M_p will be *timelike* (if $g_{ab}\xi^a\xi^b > 0$), *null* (if $g_{ab}\xi^a\xi^b = 0$), or *spacelike* (if $g_{ab}\xi^a\xi^b < 0$). Null vectors create the cone structure; timelike vectors are inside the cone while spacelike vectors are outside. A *time orientable* spacetime is one that has a continuous timelike vector field on M. A time orientable spacetime allows us to distinguish between the future and past lobes of the light cone. In what follows, it is assumed that spacetimes are time orientable.

For some interval $I \subseteq \mathbb{R}$, a smooth curve $\gamma : I \to M$ is *timelike* if the tangent vector ξ^a at each point in $\gamma[I]$ is timelike. Similarly, a curve is *null* (respectively, *spacelike*) if its tangent vector at each point is null (respectively, spacelike). A curve is *causal* if its tangent vector at each point is either null or timelike. A causal curve is *future-directed* if its tangent vector at each point falls in or on the future lobe of the light cone.

For any two points $p, q \in M$, we write $p \ll q$ if there exists a future-directed timelike curve from p to q. We write $p < q$ if there exists a future-directed causal curve from p to q. These relations allow us to define the *timelike and causal pasts and futures* of a point p: $I^-(p) = \{q : q << p\}$, $I^+(p) = \{q : p << q\}$, $J^-(p) = \{q : q < p\}$, and $J^+(p) = \{q : p < q\}$. Naturally, for any set $S \subseteq M$, define $J^+[S]$ to be the set $\cup\{J^+(x) : x \in S\}$ and so on. A *closed timelike curve* is a timelike curve $\gamma : I \to M$ such that there are distinct points $s, s' \in I$ with $\gamma(s) = \gamma(s')$.

A point $p \in M$ is a *future endpoint* of a future-directed causal curve $\gamma : I \to M$ if, for every neighborhood O of p, there exists a point $t_0 \in I$ such that $\gamma(t) \in O$ for all $t > t_0$. A *past endpoint* is defined similarly. For any set $S \subseteq M$, we define the *past domain of dependence of S* (written $D^-(S)$) to be the set of points $p \in M$ such

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[1] The reader is encouraged to consult Wald (1984) and Malament (2012) for details. An outstanding (and less technical) survey of the global structure of spacetime is given by Geroch & Horowitz (1979).

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that every causal curve with past endpoint p and no future endpoint intersects S. The *future domain of dependence of S* (written $D^+(S)$) is defined analogously. The entire *domain of dependence of S* (written $D(S)$) is just the set $D^-(S) \cup D^+(S)$.

We say a curve $\gamma : I \to M$ is not *maximal* if there is another curve $\gamma' : I' \to M$ such that I is a proper subset of I' and $\gamma(s) = \gamma'(s)$ for all $s \in I$. A curve $\gamma : I \to M$ in a spacetime (M, g_{ab}) is a *geodesic* if $\xi^a \nabla_a \xi^b = \mathbf{0}$ where ξ^a is the tangent vector and ∇_a is the unique derivative operator compatible with g_{ab}. A spacetime (M, g_{ab}) is *geodesically complete* if every maximal geodesic $\gamma : I \to M$ is such that $I = \mathbb{R}$. We say that a future-directed timelike or null geodesic $\gamma : I \to M$ without future endpoint is *future incomplete* if there is an $r \in \mathbb{R}$ such that $s < r$ for all $s \in I$. A *past incomplete* timelike or null geodesic is defined analogously.

A set $S \subset M$ is *achronal* if no two points in S can be connected by a timelike curve. The *edge* of a closed, achronal set $S \subset M$ is the set of points $p \in S$ such that every open neighborhood O of p contains a point $q \in I^+(p)$, a point $r \in I^-(p)$, and a timelike curve from r to q which does not intersect S. A set $S \subset M$ is a *slice* if it is closed, achronal, and without edge. A spacetime (M, g_{ab}) which contains a slice S such that $D(S) = M$ is said to be *globally hyperbolic*. A set $S \subset M$ is a *spacelike surface* if S is an $(n-1)$-dimensional submanifold (possibly with boundary) such that every curve in S is spacelike.

Finally, two spacetimes (M, g_{ab}) and (M', g'_{ab}) are *isometric* if there is a diffeomorphism $\psi : M \to M'$ such that $\psi_*(g_{ab}) = g'_{ab}$. We say a spacetime (M', g'_{ab}) is an *extension* of (M, g_{ab}) if there is a subset N of M' such that (M, g_{ab}) and $(N, g'_{ab|N})$ are isometric. We say a spacetime is *maximal* if it has no extension other than itself. A spacetime (M, g_{ab}) is *past maximal* if, for each of its maximal extensions (M', g'_{ab}) with isometric embedding $\psi : M \to M'$, we have $I^-(\psi(M)) = \psi(M)$. A *future maximal* spacetime is defined analogously.

3. Time machines

One wonders if closed timelike curves (CTCs), which allow for time travel of a certain type, can be "created" by rearranging the distribution and flow of matter (Stein, 1970). In other words, can a physically reasonable spacetime contain a "time machine" of sorts? Here, we examine one way of formalizing the question due to Earman, Smeenk, and Wüthrich (2009), Earman and Wüthrich (2010), and Smeenk and Wüthrich (2011).

Consider a past maximal, globally hyperbolic spacetime (M, g_{ab}). It represents a "time" before the machine is switched on.[2] We would like to capture the idea that this spacetime "creates" CTCs. Accordingly, we can require that every "physically reasonable" maximal extension of (M, g_{ab}) must contain CTCs. Consider the following statement.

> (T) There is a past maximal, globally hyperbolic spacetime (M, g_{ab}) such that every maximal extension of (M, g_{ab}) with property $\mathfrak{p}$ contains CTCs.

We seek to find physically reasonable "potency" properties $\mathfrak{p}$ which make (T) true. And we know from counterexamples constructed by Krasnikov (2002) that (T) will be false unless there *is* a potency property $\mathfrak{p}$ which limits spacetime "holes" in some sense.

Are there any properties $\mathfrak{p}$ which make (T) true? Yes. We say a spacetime (M, g_{ab}) is *hole-free* if, for any spacelike surface S in M there is no isometric embedding $\theta : D(S) \to M'$ into another spacetime (M', g'_{ab}) such that $\theta(D(S)) \neq D(\theta(S))$. It has been argued that all physically reasonable spacetimes are hole-free (Geroch, 1977; Clarke, 1976). And it turns out that (T) is true if $\mathfrak{p}$ is hole-freeness (Manchak, 2009a). The two-dimensional Misner spacetime can be used to prove the result. However, it seems that hole-freeness cannot be regarded as physically reasonable potency property after all; it turns out (!) that Minkowski spacetime is not hole-free (Krasnikov, 2009).

Because hole-freeness fails to be physically reasonable, one seeks more appropriate alternate potency properties to rule out holes. Here, we consider two such.[3] First we have (Manchak, 2014):

Definition. A spacetime (M, g_{ab}) is *E complete* if, for every future or past incomplete timelike geodesic $\gamma : I \to M$, and every open set O containing γ, there is no isometric embedding $\varphi : O \to M'$ into some other spacetime (M', g'_{ab}) such that $\varphi \circ \gamma$ has future and past endpoints.

One can show that every geodesically complete spacetime (e.g. Minkowski spacetime) is E complete (Manchak, 2014). In this sense, E completeness is a more appropriate condition than hole-freeness.

Our second condition is:

Definition. A spacetime (M, g_{ab}) is *J closed* if, for all $p \in M$, the sets $J^+(p)$ and $J^-(p)$ are closed.

One can show that every globally hyperbolic spacetime (e.g. Minkowski spacetime) is J closed (Hawking & Sachs, 1974). In this sense, J closedness is a more appropriate condition than hole-freeness.

Now we are in a position to ask two precise questions: Is (T) true when $\mathfrak{p}$ is E completeness? J closedness? Let's begin with the first question. We have:

Proposition 1. *If $\mathfrak{p}$ is E completeness,* (T) *is true.*

Proof. Let (N, η_{ab}) be a Misner spacetime. So, $N = \mathbb{R} \times \mathbb{S}$ and $\eta_{ab} = 2\nabla_{(a}t\nabla_{b)}\varphi + t\nabla_a\varphi\nabla_b\varphi$ where the points (t, φ) are identified with the points $(t, \varphi + 2\pi n)$ for all integers n. Let (M, g_{ab}) be such that $M = \{(t, \varphi) \in N : t < 0\}$ and $g_{ab} = \eta_{ab|M}$. One can verify that (M, g_{ab}) is a past maximal, globally hyperbolic spacetime.

Let (M'', g''_{ab}) be any extension whatsoever to (M, g_{ab}). For convenience, take the associated isometric embedding $\psi : M \to M''$ to be the identity function. In (M'', g''_{ab}) there will be a future incomplete timelike geodesic $\gamma : I \to M$ with past endpoint at $(-1, 0)$; the geodesic winds around the spacetime, ever approaching but never meeting $t = 0$ (Hawking & Ellis, 1973).

Let (M', g'_{ab}) be such that $M' = \mathbb{R} \times \mathbb{S}$ and $g'_{ab} = -2\nabla_{(a}t'\nabla_{b)}\varphi' + t'\nabla_a\varphi'\nabla_b\varphi'$ where the points (t', φ') are identified with the points $(t', \varphi' + 2\pi n)$ for all integers n. We know (M', g'_{ab}) is an extension of (M, g_{ab}) (Hawking & Ellis, 1973); let $\psi' : M \to M'$ be the associated isometric embedding. One can verify that $\psi' \circ \gamma$ has past endpoint at $t' = -1$ and future endpoint at $t' = 0$. Thus (M'', g''_{ab}) is not E complete. Thus, since (M'', g''_{ab}) was arbitrary, every maximal, E complete extension of (M, g_{ab}) contains CTCs. □

Now observe: in the proof above, (M, g_{ab}) renders (T) true but it does so only vacuously. There do not *exist* any maximal, E complete extensions of (M, g_{ab}); so all of them must contain CTCs. This suggests a minor alteration to (T).

> (T*) There is a past maximal, globally hyperbolic spacetime (M, g_{ab}) such that (i) there is a maximal extension of (M, g_{ab}) with property $\mathfrak{p}$ and (ii) every such extension contains CTCs.

[2] All globally hyperbolic spacetimes fail to have CTCs (Wald, 1984).

[3] For others, see Clarke (1993), Manchak (2009b), and Minguzzi (2012).

Is (T*) true when $\mathfrak{p}$ is E completeness? This is presently unknown. Is (T*) true when $\mathfrak{p}$ is J closedness? Yes. A proof, using Misner spacetime, is given in Manchak (2011). We have:

Proposition 2. *If $\mathfrak{p}$ is J closedness*, (T*) *is true*.

4. Hole machines?

Are J closedness and E completeness physically reasonable properties to impose on spacetime? In the present context, our investigation has centered upon the "no hole" properties of (presumably reasonable) past maximal, globally hyperbolic spacetimes and their maximal extensions. Therefore, it seems appropriate, at least for the time being, to require $\mathfrak{p}$ to be such that, for any past maximal, globally hyperbolic spacetime, there is *at least a chance* that there is a maximal extension of that spacetime with $\mathfrak{p}$. This idea (or something close to it) was considered by Clarke (1976), Earman (1989), and Manchak (2009b). Consider the following:

Definition. A property $\mathfrak{p}$ is *future Cauchy extendible* if, for every past maximal, globally hyperbolic spacetime (M, g_{ab}), there is a maximal extension of (M, g_{ab}) with property $\mathfrak{p}$.

We now have two more precise questions: Is E completeness future Cauchy extendible? Is J closedness? No and no. The E completeness case follows directly from the proof of Proposition 1. We have:

Proposition 3. *E completeness is not future Cauchy extendible.*

Proof. Let (M, g_{ab}) be the past maximal, globally hyperbolic portion of Misner spacetime. As outlined in Proposition 1, any extension (maximal or not) to (M, g_{ab}) fails to be E complete. □

The J closedness case was conjectured by Geroch (private communication). The result is somewhat surprising; we would expect a (globally hyperbolic and thus) J closed spacetime to have some J closed maximal extension. Instead, we have:

Proposition 4. *J closedness is not future Cauchy extendible.*

Proof. Let (N, η_{ab}) be a Misner spacetime. So, $N = \mathbb{R} \times \mathbb{S}$ and $\eta_{ab} = 2\nabla_{(a} t \nabla_{b)} \varphi + t \nabla_a \varphi \nabla_b \varphi$ where the points (t, φ) are identified with the points $(t, \varphi + 2\pi n)$ for all integers n. Now let (N', η'_{ab}) be a spacetime such that $N' = N - \{(0,0)\}$ and $\eta'_{ab} = \Omega^2 \eta_{ab}$ where $\Omega : N' \to \mathbb{R}$ approaches zero as the point $(0,0)$ is approached along the line $\{(t, \varphi) \in N' : \varphi = 0 \& t < 0\}$. Let (M, g_{ab}) be such that $M = \{(t, \varphi) \in N' : t < 0\}$ and $g_{ab} = \eta'_{ab|M}$. One can verify that (M, g_{ab}) is past maximal, globally hyperbolic.

Let (M', g'_{ab}) be any maximal extension of (M, g_{ab}). Now, for every $k \in [0, 2\pi]$, let γ_k be the null geodesic whose image is the set $\{(t, \varphi) \in M : \varphi = k \& -1 < t < 0\}$. Now, for each k, γ_k either has a future endpoint p_k or not. Let K be the set of all the endpoints p_k. Because (M', g'_{ab}) is maximal, there will certainly be some k such that the point p_k is in K. But since Ω approaches zero as the point $(0,0)$ is approached along γ_0, the point p_0 does not exist. Hence, K is not (the image of) a closed null curve. This implies that there will be some point $q \in J^+(p_k) \cap K$ such that $q \notin J^-(p_k)$. But of course, $q \in \overline{J^-(p_k)}$ since $q \in K$. So $J^-(p_k)$ is not a closed set and, therefore, (M', g'_{ab}) is not J closed. □

Given these results, a time machine skeptic might conclude that (T*) unreasonably restricts attention only to maximal extensions which satisfy $\mathfrak{p}$. After all, Propositions 3 and 4 seem to show us that (presumably) reasonable spacetimes do not always have maximal extensions which are E complete or J closed. In fact, one might argue that these results actually support the existence of "hole machines" of a certain type. Consider the following statement:

> (H) There is a past maximal, globally hyperbolic spacetime (M, g_{ab}) such that every maximal extension of (M, g_{ab}) fails to have property $\mathfrak{p}$.

One could seek "no hole" properties $\mathfrak{p}$ which make (H) true. As before, the past maximal, globally hyperbolic spacetime (M, g_{ab}) represents a "time" before the hole machine is switched on.[4] Requiring that every maximal extension of (M, g_{ab}) fail to have $\mathfrak{p}$ captures the idea that this spacetime "creates" holes. Notice that an existence clause, as in (T*), is not needed here since every spacetime has a maximal extension (Geroch, 1970). Of course, as corollaries to Propositions 3 and 4, we have:

Proposition 5. *If $\mathfrak{p}$ is E completeness*, (H) *is true*.

Proposition 6. *If $\mathfrak{p}$ is J closedness*, (H) *is true*.

We see that, if $\mathfrak{p}$ is J closedness, the time machine advocate and hole machine advocate each has a result in hand. But this is not the case if $\mathfrak{p}$ is E completeness; as noted before, it is still an open question whether (T*) is true if $\mathfrak{p}$ is E completeness. So we see one sense in which (so far) the hole machine advocate is in a better position than the time machine advocate.

But there are other (more serious) senses in which (so far) the hole machine advocate is in a better position than the time machine advocate. Consider the presuppositions made by each. Both advocates seem to presume that all past maximal, globally hyperbolic spacetimes are physically reasonable. And both presume that all physically reasonable extensions to such globally hyperbolic regions must be maximal. But the hole machine advocate requires no potency properties to deduce the existence of holes. In (H), one considers *all* maximal extensions of (M, g_{ab}); *no demarcation between the reasonable and unreasonable is presumed*.

And not only is it potentially problematic to demarcate between the reasonable and unreasonable extensions. But in fact (one might argue) the demarcations presumed (so far) are suspect. After all, Propositions 5 and 6 show a sense in which J closedness and E completeness are not satisfied by all physically reasonable spacetimes. *Why, then, should the time machine advocate be permitted to use these properties as potency properties?*[5]

5. Conclusion

What can the time machine advocate do to better her position? It seems she must reject the presumption that all past maximal, globally hyperbolic spacetimes are physically reasonable. Consider the following statements:

> (T**) There is a past-maximal, globally hyperbolic spacetime (M, g_{ab}) with property $\mathfrak{q}$ such that (i) there is a maximal extension of (M, g_{ab}) with property $\mathfrak{p}$ and (ii) every such extension contains CTCs.

> (H**) There is a past-maximal, globally hyperbolic spacetime (M, g_{ab}) with property $\mathfrak{q}$ such that every maximal extension of (M, g_{ab}) fails to have property $\mathfrak{p}$.

[4] One cannot require (M, g_{ab}) to have property $\mathfrak{p}$ since some "no hole" properties (e.g. E completeness) imply the maximality of spacetime (Manchak, 2014). But all globally hyperbolic spacetimes (M, g_{ab}) have the property of "internal causal compactness": for all points $p, q \in M$, the closure of $I^-(p) \cap I^+(q)$ is compact. This property "excludes holes" in some sense (Geroch & Horowitz, 1979, p. 251).

[5] Thanks to Thomas Barrett and Jim Weatherall for leading me to this idea.

Here, $\mathfrak{q}$ is some "local" property (e.g. an "energy condition") satisfied by physically reasonable spacetimes (Manchak, 2013). As before, $\mathfrak{p}$ is some "no holes" property. The time machine advocate can hope that, for some choices of $\mathfrak{p}$ and $\mathfrak{q}$, (T**) winds up being true and (H**) winds up being false. But it is not yet clear which of the two advocates benefits more from limiting the discussion in this way. There is more work to be done here.

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