On Gödel and the Ideality of Time

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Gödel's remarks concerning the ideality of time are examined. In the literature, some of these remarks have been somewhat neglected while others have been heavily criticized. In this article, I propose a clear and defensible sense in which Gödel's work bears on the question whether there is an objective lapse of time in our world.

1. Introduction. The cosmological model given by Gödel (1949a) is an exact solution of Einstein's equation in which matter takes the form of a pressure-free perfect fluid. Its peculiar causal properties (e.g., a global time function fails to exist) have been of considerable interest to philosophers of time since the properties seem to imply the nonexistence of an objective time lapse. But it is not clear how the peculiar features of the Gödel model bear on the nature of time in our own universe. This thought Gödel explicitly considered. He writes (1949b, 561–62): "It might, however, be asked: Of what use is it if such conditions prevail in certain *possible* worlds? Does that mean anything for the question interesting us whether in *our* world there exists an objective lapse of time?" Gödel offers two remarks in response to the questions (562):

> I think it does. For: (1) Our world, it is true, can hardly be represented by the particular kind of rotating solutions referred to above (because the solutions are static and, therefore, yield no red-shift for distant objects); there exist however also expanding rotating solutions. In such universes an absolute time might fail to exist, and it is not impossible that our world is a universe of this kind. (2) The mere compatibility with the laws of nature of worlds in which there is no distinguished absolute time . . . throws some

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> light on the meaning of time in those worlds in which an absolute time can be defined. For, if someone asserts that this absolute time is lapsing, he accepts as a consequence that whether or not an objective lapse of time exists . . . depends on the particular way in which matter and its motion are arranged in the world. This is not a straightforward contradiction; nevertheless a philosophical view leading to such consequences can hardly be considered as satisfactory.

Concerning remark 1, Gödel has been somewhat neglected (Yourgrau 1991; Savitt 1994; Earman 1995; Dorato 2002; Belot 2005; Smeenk and Wüthrich 2011). Concerning remark 2, Gödel has been heavily criticized (Savitt 1994; Earman 1995; Dorato 2002; Belot 2005; Smeenk and Wüthrich 2011). What follows is intended to be a straightforward defense of remark 1 and charitable reconstruction of remark 2 that, together, serve to clarify the significance of Gödel's work for the nature of time in our world.

2. Preliminaries. We begin with a few preliminaries concerning the relevant background formalism of general relativity.[1] An n-dimensional, relativistic *space-time* (for $n \geq 2$) is a pair of mathematical objects (M, g_{ab}), where M is a connected n-dimensional manifold (without boundary) that is smooth (infinitely differentiable). Here, g_{ab} is a smooth, nondegenerate, pseudo-Riemannian metric of Lorentz signature $(-, +, \ldots, +)$ defined on M. Two space-times (M, g_{ab}) and (M', g'_{ab}) are *isometric* if there is a diffeomorphism $\phi : M \to M'$ such that $\phi_*(g_{ab}) = g'_{ab}$.

For each point $p \in M$, the metric assigns a cone structure to the tangent space M_p. Any tangent vector ξ^a in M_p will be *time-like* (if $g_{ab}\xi^a\xi^b < 0$), *null* (if $g_{ab}\xi^a\xi^b = 0$), or *space-like* (if $g_{ab}\xi^a\xi^b > 0$). Null vectors create the cone structure; time-like vectors are inside the cone, while space-like vectors are outside. A *time orientable* space-time is one that has a continuous time-like vector field on M. A time orientable space-time allows us to distinguish between the future and past lobes of the light cone. In what follows, it is assumed that space-times are time orientable.

For some interval $I \subseteq \mathbb{R}$, a smooth curve $\gamma : I \to M$ is time-like if the tangent vector ξ^a at each point in $\gamma[I]$ is time-like. Similarly, a curve is null (respectively, space-like) if its tangent vector at each point is null (respectively, space-like). A curve is *causal* if its tangent vector at each point is either null or time-like. A causal curve is *future directed* if its tangent vector at each point falls in or on the future lobe of the light cone. A time-like curve $\gamma : I \to M$ is *closed* if there are two distinct points $s_1, s_2 \in I$ such that $\gamma(s_1) = \gamma(s_2)$.

1. The reader is encouraged to consult Wald (1984) and Malament (2012) for details.

For any two points $p, q \in M$, we write $p \ll q$ if there exists a future-directed time-like curve from p to q. We write $p < q$ if there exists a future-directed causal curve from p to q. These relations allow us to define the *time-like and causal pasts and futures* of a point p: $I^-(p) = \{q : q \ll p\}$, $I^+(p) = \{q : p \ll q\}$, $J^-(p) = \{q : q < p\}$, and $J^+(p) = \{q : p < q\}$. We say a space-time (M, g_{ab}) admits a *global time function* if there is a smooth function $t : M \to \mathbb{R}$ such that, for any distinct points $p, q \in M$, if $p \in J^+(q)$, then $t(p) > t(q)$. The function assigns a "time" to every point in M such that it increases along every (nontrivial) future-directed causal curve. Following the literature (e.g., Earman 1995; Dorato 2002), we take the existence of a global time function to be a necessary (but not sufficient) condition for the objective lapse of time.

3. Concerning Remark 1. Gödel's first consideration relating to the ideality of time and our universe is clear: despite the empirical data collected by cosmologists (e.g., data suggesting an expanding universe), there remains the epistemic possibility that our universe is one in which absolute time cannot be defined. Concerning 1, Yourgrau (1991), Savitt (1994), and Dorato (2002) are silent. Earman (1995) and Belot (2005) do consider Gödel's claim but swiftly find it unconvincing.

Belot states that Gödel himself grants that the more adequate models of our cosmos support an absolute time (2005, 270). But the text certainly does not lead us to this conclusion. And we know that, even late in his life, Gödel had still not given up on the possibility that we inhabit a Gödel-type model. Indeed, he would remain intensely interested in the collection of all astronomical data relevant to this possibility (Bernstein 1991). Earman (1995, 199) claims that "we have all sorts of . . . experiences which lend strong support to the inference that we do not inhabit a Gödel type universe but rather a universe that fulfills all of the necessary conditions for an objective lapse of time." However, Earman does leave open the possibility that Gödel's remark 1 can be defended. To do so, it is sufficient to show that "there are cosmological models that (i) lack the features necessary for an objective time lapse, but (ii) reproduce the redshift, etc., so that they are effectively observationally indistinguishable from models that fit current astronomical data and have the spatiotemporal structure needed to ground an objective lapse of time" (199).

Earman strongly doubts that one can find "models which allow for time travel and which are observationally indistinguishable from non-time travel models" (1995, 200). But this remark is puzzling since one need not find models that are so causally misbehaved as to allow for time travel—it would suffice to find models that lack some feature or other necessary for an objective time lapse. And it has already been shown by Malament (1977, 79–80) that the relation of observational indistinguishability as introduced by Glymour (1977) and used by Earman (1995) does not always preserve some properties necessary for an objective time lapse (in particular, the existence of a global time function). Also, the relation of observational indistinguishability as introduced by Glymour is symmetric and allows observers to live eternally. But, these conditions can be justifiably softened. Indeed, Malament (1977) has introduced the weaker relation: the space-time (M, g_{ab}) is *weakly observationally indistinguishable* from the space-time (M', g'_{ab}) if for every point $p \in M$ there is a point $p' \in M'$ such that $I^{-}(p)$ and $I^{-}(p')$ are isometric.

So, one might now wonder whether there are cosmological models that fit current astronomical data that are weakly observationally indistinguishable from models that lack features necessary for an objective time lapse (in particular, the existence of a global time function). And it turns out that there are. In fact, one can show that every cosmological model is weakly observationally indistinguishable from some model that lacks features necessary for an objective time lapse.

> **Proposition 1.** Every space-time (M, g_{ab}) is weakly observationally indistinguishable from a space-time (M', g'_{ab}) that fails to have a global time function.

It should be clear that the proposition (a proof is given in the appendix) provides support for Gödel's remark 1. Indeed, it remains an epistemic possibility, just as Gödel claimed it was, that we inhabit a world that has no objective time lapse. Further, even in the face of any (as yet uncollected) astronomical data, this epistemic possibility remains. One final comment concerning the proposition: it makes precise the heavily criticized statement made elsewhere by Gödel that "the experience of the lapse of time can exist without an objective lapse of time" (1949b, 561).

4. Concerning Remark 2. Gödel's second remark is sometimes interpreted to be an argument that time in our universe is ideal (Savitt 1994; Earman 1995; Dorato 2002). But this reading seems to be a bit strong. Gödel only states that "the mere compatibility with the laws of nature of worlds in which there is no distinguished absolute time . . . throws some light on the meaning of time in those worlds in which an absolute time can be defined." In other words, the existence of Gödel-type solutions simply have implications regarding the nature of time for all cosmological models. However, it seems "there is a consensus that even this modest conclusion is not warranted" (Smeenk and Wüthrich 2011, 597).

Now we have already shown that the nonexistence of a global time function in certain models does have epistemic implications for all models. But Gödel seems to have something more in mind concerning remark 2. Indeed, of great importance seems to be that fact that "whether or not an objective lapse of time exists . . . depends on the particular way in which matter and its motion are arranged in the world." And it is unclear how this statement leads to any general implications concerning all models. Here, we provide one way to spell out some of the details.

Consider an arbitrary model (M, g_{ab}) that has all the geometric properties necessary for absolute time. In particular, assume it admits a global time function. Following Gödel, consider an observer at some point $p \in M$ who "asserts that this absolute time is lapsing." Since time is lapsing objectively, this means that at p—the event of the assertion—all events in the set $I^+(p)$ have yet to "come into existence" (Gödel 1949b, 558). In other words, at p, if one takes the idea of an objective time lapse seriously, one is led to consider as fixed not the space-time (M, g_{ab}) but rather merely a portion of it. It is then natural to wonder whether there is a sense in which this leaves open from the perspective of the observer making the assertion the nomological possibility that, after the time of the assertion, matter and its motions might be (re)arranged in such a way that a global time function can no longer be defined. This question whether a cosmological model can "start out" with well-behaved causal structure but not "end up" that way was, in a sense, asked some time ago by Stein (1970, 594). "Consider either an arbitrary given cosmological model, or a model having the structure of one of the sorts assumed to hold in the real world. Then: is it ((a) ever, (b) always) possible to introduce into such a model a continuous deformation of the structure, leading through intermediate states, all compatible with Einstein's theory, to a state in which Gödel-type relationships occur?"

Here is one way to formulate this question precisely. Let (M, g_{ab}) be any space-time that admits a global time function. For any point $p \in M$, is there is a space-time (M, g'_{ab}) that (i) fails to admit a global time function and (ii) is such that $g'_{ab} = g_{ab}$ on the region $M - I^+(p)$? When (M, g_{ab}) is at least three-dimensional, yes.

> **Proposition 2.** Let (M, g_{ab}) be any space-time of dimension $n \geq 3$ that admits a global time function. For any point $p \in M$, there is a space-time (M, g'_{ab}) that (i) fails to admit a global time function and (ii) is such that $g'_{ab} = g_{ab}$ on the region $M - I^+(p)$.

It should be clear how the proposition (proof given in the appendix) can be used to understand Gödel's remark 2. It is philosophically unsatisfying (although not a contradiction) for one to assert that time is objectively lapsing in one's universe when from the perspective of the observer making the assertion there remains the nomological possibility that, after the time of the assertion, matter and its motions might be smoothly (re)arranged in such a way so as to prohibit an objective time lapse.

5. Conclusion. Despite the above propositions, one might insist that "we do not inhabit a Gödel type universe but rather a universe that fulfills all of the necessary conditions for an objective lapse of time" (Earman 1995, 199). We close with one final word of caution. It seems reasonable that "in order to be physically significant, a property of space-time ought to have some form of stability, that is to say, it should also be a property of 'nearby' space-times" (Hawking and Ellis 1973, 197). There are a number of different ways to understand the notion of nearby space-times—none entirely satisfactory (Geroch 1971; Fletcher 2015). Here we simply note one sense in which causally well-behaved space-times can be "close" to space-times that are not.

Consider the one-parameter family of space-times $(M, g_{ab}(\lambda))$ where $\lambda \in [0, 1]$, $M = \mathbb{R}^4$, and $g_{ab}(\lambda) = -\nabla_a t \nabla_b t + \nabla_a x \nabla_b x - (1/2)\exp(2\lambda x) \nabla_a y \nabla_b y - 2\exp(\lambda x) \nabla_{(a} t \nabla_{b)} y + \nabla_a z \nabla_b z$. One can easily verify that $(M, g_{ab}(\lambda))$ is Gödelian for all $\lambda \in (0, 1]$. But what about $(M, g_{ab}(0))$? Surprisingly, one finds it is a Minkowski space-time.[2] Thus, there is sense in which a model satisfying all of the necessary conditions for an objective lapse of time is "close" to a set of models that do not satisfy these conditions. Should this fact not give us pause?

Appendix

Lemma 1. Let (M, g_{ab}) be any space-time and let O be any open set in M. There is an open set $\hat{O}$ in O and a space-time (M, g'_{ab}) such that g'_{ab} is flat on $\hat{O}$ and $g'_{ab} = g_{ab}$ on $M - O$.

Proof. Let (M, g_{ab}) be any two-dimensional space-time (one can generalize to higher dimensions), and let O be any open set in M. Consider a chart (O', φ) such that (i) $O' \subset O$; (ii) for some $\epsilon > 0$, $\varphi[O']$ is the open ball $B_\epsilon(0, 0)$ centered at the origin in $\mathbb{R}^2$ with radius ϵ; and (iii) the coordinate maps $t : O' \to \mathbb{R}$ and $x : O' \to \mathbb{R}$ associated with (O', φ) are such that g_{ab} at the point $\varphi^{-1}(0, 0)$ is $-\nabla_a t \nabla_b t + \nabla_a x \nabla_b x$. We can now express $g_{ab|O'}$ as $f \nabla_a t \nabla_b t + g \nabla_a x \nabla_b x + 2h \nabla_{(a} t \nabla_{b)} x$ for some smooth scalar fields $f : O' \to \mathbb{R}$, $g : O' \to \mathbb{R}$, and $h : O' \to \mathbb{R}$.

Let $\eta_{ab} = f' \nabla_a t \nabla_b t + g' \nabla_a x \nabla_b x + 2h' \nabla_{(a} t \nabla_{b)} x$ be a flat (Lorentzian) metric on O' for some smooth scalar fields $f' : O' \to \mathbb{R}$, $g' : O' \to \mathbb{R}$, and $h' : O' \to \mathbb{R}$ such that η_{ab} at the point $\varphi^{-1}(0, 0)$ is $-\nabla_a t \nabla_b t + \nabla_a x \nabla_b x$.

Since $f = f' < 0 < g = g'$ at the point $\varphi^{-1}(0, 0)$, we can find a $\delta \in (0, \epsilon)$ such that $f < 0 < g$ and $f' < 0 < g'$ on all of $\varphi^{-1}[B_\delta(0, 0)]$. Let $O'' \subset O'$ be this set $\varphi^{-1}[B_\delta(0, 0)]$. Now we divide O'' into three disjoint regions: U, V, W. For convenience, let r be the scalar function on O'' defined by $\sqrt{t^2 + x^2}$. Let U be the region where $r < \delta/3$; V the region where $\delta/3 \le r < 2\delta/3$; W the region where $2\delta/3 \le r < \delta$.

2. Thanks to David Malament for this example.

Next, we define a field γ_{ab} on each of the three regions. On region W, let $\gamma_{ab} = g_{ab}$. On region U, let $\gamma_{ab} = \eta_{ab}$. On region V, let γ_{ab} be as follows: $(\theta f + (1-\theta)f')\,\nabla_a t\,\nabla_b t + (\theta g + (1-\theta)g')\,\nabla_a x\,\nabla_b x + 2(\theta h + (1-\theta)h')\,\nabla_{(a} t\,\nabla_{b)} x$, where $\theta : V \to \mathbb{R}$ is given by

$$\theta(r) = \frac{\int_0^{3(r-\delta/3)/\delta} \exp[-(z^{-2} + (z-1)^{-2})]dz}{\int_0^1 \exp[-(z^{-2} + (z-1)^{-2})]dz}.$$

By inspection, one can see that γ_{ab} is smooth field on O'' (cf. Geroch 1968, 536). We will work to show that it is a metric. Clearly, it is everywhere symmetric and is nondegenerate on U and W. We claim it is nondegenerate on V as well. For convenience, let $f'' = \theta f + (1-\theta)f'$, $g'' = \theta g + (1-\theta)g'$, $h'' = \theta h + (1-\theta)h'$. Let p be any point in V, and let ξ^a be any vector at p. We can express ξ^a as $\alpha(\partial/\partial t)^a + \beta(\partial/\partial x)^a$ for some $\alpha, \beta \in \mathbb{R}$. Consider $\gamma_{ab}\xi^a$. It must come out as $(f''(p)\alpha + h''(p)\beta)\,\nabla_b t + (h''(p)\alpha + g''(p)\beta)\,\nabla_b x$. Now suppose that $\gamma_{ab}\xi^a = 0$. This implies that $f''(p)\alpha + h''(p)\beta = 0$ and $h''(p)\alpha + g''(p)\beta = 0$. It follows that $\alpha(f''(p)g''(p) - h''(p)^2) = 0$ and $\beta(f''(p)g''(p) - h''(p)^2) = 0$. So either $\alpha = \beta = 0$ or $f''(p)g''(p) = h''(p)^2$. But the latter case cannot obtain: because $f(p) < 0 < g(p)$, $f'(p) < 0 < g'(p)$, and $\theta(p) \in [0,1]$, we know $f''(p) < 0 < g''(p)$. So $\alpha = \beta = 0$, and thus $\xi^a = 0$. So γ_{ab} is nondegenerate on V. So, γ_{ab} is a smooth metric on O''. Since γ_{ab} is Lorentzian at $\varphi^{-1}(0,0)$ and O'' is connected, γ_{ab} is Lorentzian on all of O''.

Now, consider the space-time (M, g'_{ab}), where $g'_{ab} = g_{ab}$ on $M - O''$ and $g'_{ab} = \gamma_{ab}$ on O''. By construction, g'_{ab} is smooth. Also by construction, there is an open set $\hat{O}$ in O such that g'_{ab} is flat on $\hat{O}$. Just take $\hat{O} = U$. QED

Lemma 2. Let (M, g_{ab}) be any space-time of dimension $n \geq 3$ and let O be any open set in M. There is a space-time (M, g'_{ab}) such that there are closed time-like curves contained in O and $g'_{ab} = g_{ab}$ on $M - O$.

Proof. Let (M, g_{ab}) be any three-dimensional space-time (one can generalize to higher dimensions) and let O be any open set in M. By the lemma above, there is an open set $\hat{O}$ in O and a space-time (M, g''_{ab}) such that g''_{ab} is flat on $\hat{O}$ and $g''_{ab} = g_{ab}$ on $M - O$.

Next, consider a chart (U, φ) such that (i) $U \subset \hat{O}$ for some $\delta > 0$, (ii) $\varphi[U]$ is the open ball $B_\delta(0,0,0)$ centered at the origin in $\mathbb{R}^3$ with radius δ, and (iii) the coordinate maps $t : U \to \mathbb{R}$, $x : U \to \mathbb{R}$, and $y : U \to \mathbb{R}$ associated with (U, φ) are such that the (flat) metric g''_{ab} on U can be expressed as the (flat) metric: $-\nabla_a t\,\nabla_b t + \nabla_a x\,\nabla_b x - (1/2)\,\nabla_a y\,\nabla_b y - 2\,\nabla_{(a} t\,\nabla_{b)} y$.

Now we divide U into three disjoint regions: U_1, U_2, U_3. For convenience, let r be the scalar function on U defined by $\sqrt{t^2 + x^2 + y^2}$. Let

U_1 be the region where $r < \delta/3$; U_2 the region where $\delta/3 \leq r < 2\delta/3$; U_3 the region where $2\delta/3 \leq r < \delta$.

Next, we define a metric g'_{ab} on M with the desired properties. On regions U_3 and $M - U$, let $g'_{ab} = g''_{ab}$. On region U_1, let g'_{ab} be Gödelian: $-\nabla_a t \nabla_b t + \nabla_a x \nabla_b x - (1/2)\exp(2ax) \nabla_a y \nabla_b y - 2\exp(ax) \nabla_{(a} t \nabla_{b)} y$, where $a > 0$ is large enough that closed time-like curves exist in U_1. On region U_2, let g'_{ab} be as follows: $-\nabla_a t \nabla_b t + \nabla_a x \nabla_b x - (1/2)\exp(2ax(1-\theta)) \nabla_a y \nabla_b y - 2\exp(ax(1-\theta)) \nabla_{(a} t \nabla_{b)} y$, where $\theta : U_2 \to \mathbb{R}$ is just the function $\theta(r)$ given in the proof of the above lemma. By inspection, one can see that g'_{ab} is a smooth metric on M (cf. Geroch 1968, 536). By construction, the space-time $M - U$ is such that there are closed time-like curves contained in O and $g'_{ab} = g_{ab}$ on $M - O$. QED

Proposition 1. Every space-time (M, g_{ab}) is weakly observationally indistinguishable from a space-time (M', g'_{ab}) that fails to have a global time function.

Proof. Let (M, g_{ab}) be a two-dimensional space-time (one can generalize to higher dimensions). If there is a point $p \in M$ such that $I^-(p) = M$, then (M, g_{ab}) fails to have a global time function. Suppose there does not exist a $p \in M$ for which $I^-(p) = M$. Construct the space-time (M', g'_{ab}) according to the method outlined in Manchak (2009). Next, consider any open set O in the $M(1, \beta)$ portion of the manifold M' that is disjoint from the set $O_1 \cup O_2$. From the first lemma, there is an open set $\hat{O}$ in O and a space-time (M', g''_{ab}) such that g'_{ab} is flat on $\hat{O}$ and $g'_{ab} = g_{ab}$ on $M - O$. Consider a chart (O', φ) such that (i) $O' \subset \hat{O}$; (ii) for some $\epsilon > 0$, $\varphi[O']$ is the open ball $B_\epsilon(0,0)$ centered at the origin in $\mathbb{R}^2$ with radius ϵ; and (iii) the coordinate maps $t : O' \to \mathbb{R}$ and $x : O' \to \mathbb{R}$ associated with (O', φ) are such that g''_{ab} is $-\nabla_a t \nabla_b t + \nabla_a x \nabla_b x$. Now, excise two sets of points from O: $S_+ = \{(t, x) : t = \epsilon/2, -\epsilon/2 \leq x \leq \epsilon/2\}$ and $S_- \{(t, x) : t = -\epsilon/2, -\epsilon/2 \leq x \leq \epsilon/2\}$. Identify the bottom edge of S_- with the top edge of S_+ and the top edge of S_- with the bottom edge of S_+ (cf. Hawking and Ellis 1973, 58–59). The resulting space-time, call it (M'', g''_{ab}), contains closed time-like curves. By construction, (M, g_{ab}) is weakly observationally indistinguishable from (M'', g''_{ab}). Of course, the nonexistence of a global time function follows from the existence of closed time-like curves. QED

Proposition 2. Let (M, g_{ab}) be any space-time of dimension $n \geq 3$ that admits a global time function. For any point $p \in M$, there is a space-time (M, g'_{ab}) that (i) fails to admit a global time function and (ii) is such that $g'_{ab} = g_{ab}$ on the region $M - I^+(p)$.

Proof. Let (M, g_{ab}) be any space-time of dimension $n \geq 3$ that admits a global time function. Let p be any point in M. By the second lemma, we know (since $I^+(p)$ is an open set) there exists a space-time (M, g'_{ab}) such that there are closed time-like curves in $I^+(p)$ and $g_{ab} = g'_{ab}$ on $M - I^+(p)$. Of course, the nonexistence of a global time function follows from the existence of closed time-like curves. QED

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