

Dedekind, Structuralism, and the Philosophy of Mathematics

Seminar, LPS, UC Irvine, Winter 2008

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DESCRIPTION:

Structuralist views have become a widely discussed topic in the philosophy of mathematics. Richard Dedekind (1831-1916) is often mentioned as a precursor of, and an influence on, current proponents of structuralism; arguably, his position also constitutes an attractive alternative to theirs. In addition, Dedekind made several technical contributions to the foundations of mathematics that have to be taken into account by any position in the philosophy of mathematics, structuralist or not. Dedekind's works are thus relevant to contemporary research both in a historical and a systematic sense. This class has three goals: First, to clarify the sense in which Dedekind held structuralist views, by studying his contributions to the foundations of arithmetic, analysis, and set theory in some detail; second, to discuss the strengths and weaknesses of his position, relative to contemporary alternatives; and third, to explore the relationship of his foundational contributions with his other mathematical works, another topics that has started to attract serious attention in the literature recently. Along the way a number of issues central to the history and philosophy of mathematics, also beyond Dedekind, will be discussed.

TEXTS:

- (1) R. Dedekind: *Essays on the Theory of Numbers*, W. Beman, ed. and ts., Dover, 1963.
- (2) R. Dedekind: *Theory of Algebraic Integers*, J. Stillwell, ed. and ts., Cambridge University Press, 1996.
- (3) A selection of secondary literature, in the form of Xerox copies (see the syllabus).

PREREQUISITES:

Background knowledge in three areas will be helpful: set theory (naïve set theory is enough); number theory (basic notions and techniques, especially in algebraic number theory); the philosophy of mathematics (logical and metaphysical issues in arithmetic, analysis, and set theory). However, none of this is a strict prerequisite; the material will be discussed in such a way that the relevant concepts and techniques are introduced as we go along. Consequently, the first three quarters of the class can be viewed as a general introduction to the philosophy of mathematics, presented from a historical point of view.

REQUIREMENTS:

The seminar will consist of a mixture of lectures (by the instructor) and class discussions (based on presentations by students). The class grade will be determined as follows:

- a) A term paper (12-15 pages), due during Finals Week—70% of the grade.
- b) Oral presentation of a piece of secondary literature—20% of the grade.
- c) Class participation (beyond the oral presentation)—10% of the grade.

SYLLABUS AND READINGS:

Week 1: Traditional Views on Numbers and Magnitudes.

Readings: E. Reck, "Frege, Natural Numbers, and Arithmetic's Umbilical Cord" (2003); R. Thiele, "The Greek Concept of Number and Magnitude" (Ch. 1 of H. Jahnke, ed., *A History of Analysis*, 2003).

Weeks 2-3: Dedekind on Continuity and the Real Numbers.

Readings: R. Dedekind, "Continuity and Irrational Numbers" (1872/1963); J. Ferreirós, "The Real Number System" (Ch. 4 of *Labyrinth of Thought*, 1999); R. Cooke, "R. Dedekind, Stetigkeit und Irrationale Zahlen" (2005).

Weeks 4-5: Dedekind on Sets, Functions, and the Natural Numbers.

Readings: R. Dedekind, "The Nature and Meaning of Numbers" (1888/1963); J. Ferreirós, "Sets and Maps as a Foundation for Mathematics" (Ch. 7 of 1999); J. Ferreirós, "R. Dedekind and G. Peano, Booklets on the Foundations of Arithmetic" (2005).

Week 6: Evaluating Dedekind's Structuralism, Part 1.

Readings: Selections from Dedekind's correspondence (1889-90); E. Reck, "Dedekind's Structuralism: An Interpretation and Partial Defense" (2003); W. Sieg & D. Schlimm: "Dedekind's Analysis of Number: Systems and Axioms" (2005).

Weeks 7-9: Dedekind's Theory of Ideals in Algebraic Number Theory.

Readings: Selections from Dedekind's *Theory of Algebraic Integers* (1877/1996); J. Stillwell, "Translator's Introduction" (for *Theory of Algebraic Integers*, 1996); H. Stein, "*Logos*, Logic, *Logistiké*: Some Philosophical Remarks on Nineteenth-Century Transformations in Mathematics" (1986); J. Ferreirós, "Dedekind and the Set-theoretic Approach to Algebra" (Ch. 3 of 1999); J. Avigad, "Methodology and Metaphysics in the Development of Dedekind's Theory of Ideals" (2006).

Week 10: Evaluating Dedekind's Structuralism, Part 2.

Readings: E. Reck, "Dedekind's Contributions to the Foundations of Mathematics" (forthcoming); E. Reck, "Dedekind, Structural Reasoning, and Mathematical Understanding" (forthcoming).

OPTIONAL READINGS:

Corry, Leo (2004): "Richard Dedekind: Numbers and Ideals" (Ch. 2 of *Modern Algebra and the Rise of Mathematical Structures*, 2nd edition).

Edwards, Howard (1980): "The Genesis of Ideal Theory".

Epple, Moritz (2003): "The End of the Science of Quantity: Foundations of Analysis, 1860-1910" (Ch. 10 of H. Jahnke, ed., *History of Analysis*).

McCarty, David (1995): "The Mysteries of Richard Dedekind".

Reck, E. & Price, M. (2000): "Structures and Structuralism in Contemporary Philosophy of Mathematics".

Reed, David (1995): "Number Theory in the Nineteenth Century" (Ch. 4 of *Figures of Thought: Mathematics and Mathematical Texts*).

Stein, Howard (1999): "Eudoxos and Dedekind: On the Ancient Greek Theory of Ratios and its Relation to Modern Mathematics".