Incompleteness: Homework 2

Due in class on Thursday, May 15

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1. When calling a set A "decidable", we frequently let the background context supply which superset $B \supseteq A$ that A is decidable with respect to. Awareness of which set B is in question is important, though. Show that there exist subsets A, B, and C of ω such that A is decidable with respect to B, but A is not decidable with respect to C.

2. Use the function $f(a, b) = 1/2(a + b)(a + b + 1) + a$ to construct a computable function from $Seq(\omega) \rightarrow \omega$. Be sure to show your function is in fact injective, so that you can effectively translate a sequence into a number and back again. You may appeal to the fact that f is bijective, which was shown in class.

3. Construct a Turing machine for one of the following:

- d(i j) = 1 if i = j, and = 0 if $i \neq j$.
- n!

4. The binomial coefficient is defined as: $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ when $k \leq n$, and 0 when $k > n$. Construct a recursive function that computes this function. (You may correctly assume that this function yields a natural number for any natural number inputs.)

5. Prove that: $E \vdash \forall x(x \lessdot \overline{S}\overline{n} \equiv (x \approx \overline{0} \vee \cdots \vee x \approx \overline{n}))$ (Problem 20.6 in the notes)

6. Let σ be a sentence of the language of arithmetic that has the form $\theta\psi$, where ψ is a quantifier-free formula, and θ is a string of zero or more existential quantifiers (e.g., $\theta = \exists x_1 \ldots \exists x_n$, for some $n \in \omega$). Show that if $\mathcal{N} \models \sigma$, then $E \vdash \sigma$.

7. Is $\{n : P_n$ is not total$\}$ effectively enumerable?